

Innovation in mathematics instruction to improve logical reasoning in High School Students

Innovación en la enseñanza de las matemáticas para mejorar el razonamiento lógico en Estudiantes de Bachillerato

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Abstract

This study analyzes how innovative educational strategies can improve logical-mathematical reasoning in first and third year high school students at the Juan de Salinas Educational Unit in Quito, Ecuador. The main objective is to strengthen problem-solving skills, critical thinking and autonomy, overcoming rote learning and improving performance in mathematics, an area with low results in international tests such as PISA. The methodology used is based on the theories of Ausubel, Piaget and Vygotsky, combined with cooperative learning and information and communication technologies (ICT) such as GeoGebra. Interactive activities, hands-on workshops and collaborative dynamics were implemented to connect mathematical concepts with real-world situations.

Key words: Educational innovation, logical reasoning, meaningful learning, ICT, mathematics, PISA tests.

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Resumen

Este estudio analiza cómo estrategias educativas innovadoras pueden mejorar el razonamiento lógico-matemático en estudiantes de primero y tercero de bachillerato de la Unidad Educativa Juan de Salinas, en Quito, Ecuador. El objetivo principal es fortalecer habilidades de resolución de problemas, pensamiento crítico y autonomía, superando el aprendizaje memorístico y mejorando el rendimiento en matemáticas, un área con resultados bajos en pruebas internacionales como PISA. La metodología empleada se basa en teorías de Ausubel, Piaget y Vygotsky, combinadas con aprendizaje cooperativo y tecnologías de la información y comunicación (TIC) como GeoGebra. Se implementaron actividades interactivas, talleres prácticos y dinámicas colaborativas para conectar los conceptos matemáticos con situaciones del mundo real.

Palabras clave: Innovación educativa, razonamiento lógico, aprendizaje significativo, TIC, matemáticas, pruebas PISA.

Introduction

Worldwide, and particularly in Ecuador, academic performance in mathematics has been one of the major educational concerns due to its direct impact on the development of critical skills necessary for academic and professional success. Mathematics, as a fundamental discipline, not only develops logical reasoning skills, but also fosters complex problem solving and analytical thinking, essential skills in the 21st century work environment. However, statistics from the National Institute for Educational Evaluation (INEVAL) show that a significant proportion of Ecuadorian students fail to achieve adequate levels of mathematical reasoning, which has a negative impact on their overall performance and limits their access to higher education.

These deficiencies affect their preparation for national exams and, therefore, their future academic and professional opportunities. The problem of low mathematics achievement is not unique to Ecuador. Globally, assessments such as the Program for International Student Assessment (PISA) have shown that numerous education systems face difficulties in equipping students with advanced, contextualized math skills that are relevant to modern life. These gaps are particularly pronounced in education systems with structural constraints, such as lack of adequate teacher training, insufficient resources, and outdated teaching methods. This

underscores the urgent need to develop and implement innovative pedagogical strategies that transform the way this crucial discipline is taught and learned.

In this context, this project was developed in the Juan de Salinas Educational Unit, located in Sangolquí, Ecuador, with the purpose of transforming traditional pedagogical practices. The institution, aware of the challenges faced by its students, sought to promote more active and meaningful learning, adapted to current needs.

The project is based on a constructivist approach, which places the student at the center of the learning process and encourages the active construction of knowledge based on concrete experiences. The proposal incorporates technological tools such as GeoGebra, which allows visualizing and solving complex mathematical problems interactively, and active methodologies such as cooperative learning, which emphasizes teamwork and joint problem solving.

This article details the theoretical foundations of the project, based on educational theories such as Piaget's and Vygotsky's constructivism and Ausubel's meaningful learning. It also explores methodological strategies designed to overcome traditional barriers in the teaching of mathematics, such as the lack of connection with everyday life and the predominant use of rote approaches. In addition, the results obtained during the implementation of the project are presented, which show significant improvements in students' motivation, participation and mathematical logical reasoning skills.

The importance of this project lies not only in the progress achieved at the Juan de Salinas Educational Unit, but also in its potential to serve as a replicable model in other educational contexts. By addressing both structural limitations and pedagogical practices, this initiative highlights the need to prioritize educational innovation, especially in high-need contexts. In doing so, it seeks to ensure that students not only acquire advanced mathematical competencies, but also develop confidence in their abilities and a resilient mindset in the face of academic and professional challenges,

Educational innovation refers to the implementation of significant changes in the teaching and learning processes, with the objective of improving educational quality, these changes may include new methodologies, technological tools, pedagogical approaches and strategies that encourage the active participation of students. In the

context of mathematics, educational innovation seeks to transform the way students interact with mathematical concepts, promoting a more dynamic and applied learning (Area-Moreira, 2020, p. 15). Constructivism, based on the theories of Jean Piaget and Lev Vygotsky, emphasizes the importance of the environment and social interaction in cognitive development (Coll, 1990, p. 67). Piaget emphasizes that learning arises from the interaction of the individual with his or her environment, which allows students to develop skills through active exploration. For his part, Vygotsky highlights the role of the social context and the use of mediators, such as language or technological tools, in learning.

Cooperative learning expands this perspective by organizing work in heterogeneous groups, where each member contributes his or her skills and experiences (Johnson & Johnson, 1999, p. 23). This dynamic promotes not only the acquisition of knowledge, but also the development of interpersonal skills such as communication, negotiation and leadership. In the mathematics classroom, cooperative learning fosters joint problem solving, allowing students to exchange ideas, clarify concepts and strengthen their understanding (Torrego, 2018, p. 56).

The Finnish educational model has established itself as an international benchmark thanks to its student-centered approach, the personalization of learning and the use of active methodologies, such as problem-based learning, these strategies encourage not only the acquisition of knowledge, but also the development of critical and logical-mathematical thinking. In this context, the integration of similar practices in the baccalaureate can enhance logical reasoning through interdisciplinary activities, collaborative projects and the use of educational technologies (García, 2019, p. 78).

Mathematical didactics focuses on the design of pedagogical strategies that facilitate deep learning of mathematical concepts, while promoting the development of logical reasoning (Díaz Godino, 2004, p. 45). Through structured activities, such as problem solving and exploration of mathematical patterns, students develop skills to analyze, abstract and connect concepts, which is essential for their academic and personal formation (Rico, 2015, p. 78).

Logical reasoning is a fundamental cognitive skill that allows students to analyze, synthesize, and evaluate information critically. In mathematics, logical reasoning is essential for solving problems, identifying patterns, and constructing valid arguments. Fostering

this skill not only improves performance in mathematics, but also prepares students to face challenges in other areas of knowledge and in everyday life (Johnson-Laird, 2010, p. 627).

Algorithm design is a powerful tool to improve logical-mathematical reasoning in high school students. This practice fosters sequential thinking by forcing students to break complex problems into orderly and coherent steps (Rico, 2015, p. 56). In addition, it promotes critical thinking by identifying patterns and abstracting key concepts. It also allows them to apply problem-solving skills in practical and real contexts, strengthening the relationship between mathematics and the world around them (Sánchez, 2018, p. 34).

Meaningful learning, proposed by David Ausubel, occurs when students relate new knowledge to previous concepts, thus building a deep and lasting understanding. In mathematics, this approach is crucial for students not only to memorize formulas, but to understand the underlying principles and be able to apply them in different contexts (Ausubel, 1968, p. 48). The use of strategies that promote meaningful learning, such as solving real problems and connecting to everyday situations, is critical to improving achievement.

Meaningful learning, proposed by David Ausubel, holds that knowledge is assimilated more effectively when new concepts are integrated into the learner's existing cognitive structures. This approach contrasts with mechanical learning, in which data is memorized without deep understanding or context.

In the mathematical field, this theory allows students to relate abstract concepts, such as functions or equations, with previous experiences or practical applications, facilitating a lasting understanding and a greater ability to solve problems (Díaz Godino, 2004, p. 23). Mathematics teaching, based on meaningful learning, focuses on building connections between ideas, promoting learning that is both functional and transformative (Chamorro, 2010, p. 12).

Information and Communication Technologies (ICT)

ICTs have revolutionized education by offering tools that facilitate access to information, collaboration and personalization of learning, in mathematics, the use of educational software, simulations, interactive platforms and digital resources can make learning more engaging and effective. In addition, ICTs allow

students to explore mathematical concepts in a visual and dynamic way, which favors comprehension and logical reasoning (UNESCO, 2017, p. 23).

The impact of ICT in mathematics education has been widely recognized for its ability to transform learning from a static experience to an interactive and dynamic one. Tools such as GeoGebra allow students to experiment with mathematical concepts in real time, observe their effects, and understand the relationships between different representations (Garcia, 2019, p. 78). These technologies not only facilitate the teaching of complex topics, but also stimulate curiosity and interest, increasing student participation and engagement (Puig et al., 2020, p. 89).

The PISA tests, developed by the Organisation for Economic Co-operation and Development (OECD), assess students' competencies in key areas such as mathematics, science and reading. In mathematics, PISA not only measures students' ability to solve traditional problems, but also their ability to apply mathematical concepts in real and complex contexts. An innovative approach to teaching mathematics, focusing on logical reasoning and meaningful learning, can contribute to improved results in these tests (OECD, 2019, p. 45).

The integration of educational innovation, logical reasoning, meaningful learning, and ICT in mathematics education has the potential to transform the learning experience of students, by using innovative methodologies and technological tools, educators can create learning environments that foster curiosity, creativity, and problem solving. This not only improves mathematics achievement, but also prepares students for the challenges of the 21st century (Hernández-Rojas, 2018, p. 12).

Materials and methods

The present study adopted a mixed methodological approach, combining quantitative and qualitative research techniques to comprehensively assess the impact of the project. This approach was chosen because of its ability to capture both measurable outcomes and the subjective perceptions of participants, providing a more holistic understanding of the effects of the strategies implemented. The three main phases of the project, detailed below, included an initial diagnosis, the design and implementation of

innovative pedagogical strategies, and a rigorous evaluation and monitoring process.

Phase 1: Initial diagnosis. The initial phase of the project focused on identifying the main weaknesses in the students' mathematical logical reasoning and on understanding the existing pedagogical practices in the Juan de Salinas Educational Unit. This stage included the following activities:

Application of surveys and diagnostic tests: Specific tests were designed and administered to measure the students' level of mathematical logical reasoning. These tests evaluated skills such as problem solving, critical thinking and understanding of abstract concepts. In addition, surveys were administered to collect data on students' perceptions of mathematics, including their levels of motivation, confidence and attitude towards the subject.

Interviews with teachers: Structured and semi-structured interviews were conducted with teachers to identify the teaching methodologies used, common challenges faced in the classroom, and teachers' training needs. These interviews revealed a strong reliance on traditional methods, with an emphasis on memorization and mechanical problem solving, and a low incorporation of technological tools.

Analysis of academic data: An exhaustive review of the students' academic records and the results obtained in national tests such as INEVAL was carried out. This analysis allowed us to identify trends and correlations between current teaching methods and low performance in mathematics.

Phase 2: Design and implementation of strategies. Based on the findings of the initial diagnosis, innovative pedagogical strategies were designed and implemented to address the critical areas identified. This phase was characterized by a focus on teacher training, reorganization of classroom practices and the introduction of active methodologies and technological tools.

Teacher training workshops: Intensive workshops were conducted to train teachers in the use of GeoGebra and other digital tools designed to facilitate the teaching of complex mathematical concepts. In addition, the workshops included training in active methodologies, such as problem-based learning (PBL) and cooperative learning, to promote a more dynamic and participatory approach in the classroom.

Reorganization of classroom practices: Traditional classes were transformed into interactive learning environments, where students actively participated in solving problems related to real-world situations. The use of practical and experimental activities was promoted to connect mathematical concepts with concrete applications, fostering a deeper and more meaningful understanding.

Cooperative learning strategies: Students worked in heterogeneous groups, which facilitated the exchange of ideas, the development of social skills and collaborative problem solving. Group dynamics were designed to encourage equal participation, ensuring that each team member contributed to the achievement of common goals.

Phase 3: Evaluation and monitoring. The final phase of the project focused on measuring the impact of the strategies implemented and gathering feedback from participants to identify opportunities for improvement. Activities carried out in this phase included:

Post-tests: Post-tests were designed and administered to assess progress in students' mathematical logical reasoning skills. These tests were aligned with the project objectives and allowed comparison of the results with the initial data. The results showed a significant improvement in students' performance, with notable increases in their ability to solve problems and apply mathematical concepts in a practical way.

Interviews and focus groups: Interviews were conducted with students and teachers to explore their perceptions of the project experience, identifying strengths and areas for improvement of the strategies implemented. The focus groups provided a space for open discussions, where participants shared their opinions and suggestions on the impact of the project on their teaching and learning processes.

Comparative data analysis: Post-test results were compared with initial data to assess the degree of improvement achieved. Qualitative data collected from interviews and surveys were also analyzed to identify relevant patterns and trends. Overall, this methodological approach made it possible not only to evaluate the effectiveness of pedagogical strategies, but also to adapt and continuously improve teaching practices according to the specific needs and contexts of students and teachers. The findings of this research highlight the importance of an integrated approach that combines quantitative and qualitative methods to address contemporary educational challenges.

Results

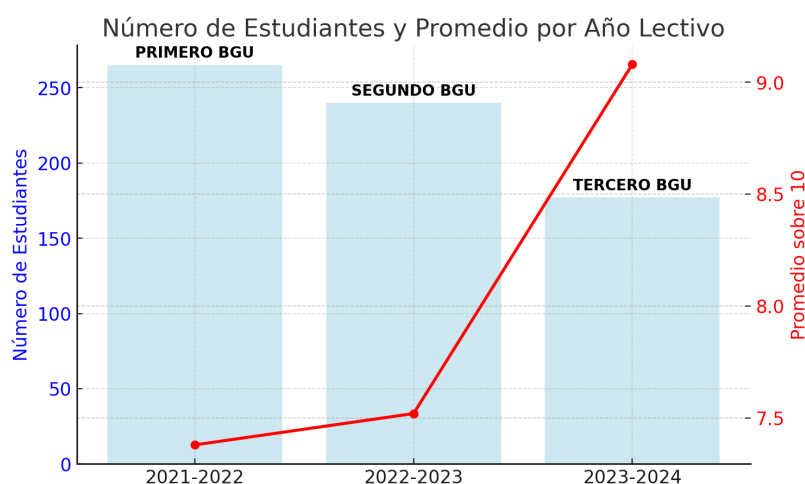
PISA tests assess 15-year-old students to determine their competencies in problem solving and the application of knowledge in real-life situations. In the case of Ecuador, the results obtained in the PISA-D 2017 tests reveal significant challenges in the area of mathematics.

This table reflects the academic performance in mathematics during the last school years, allowing an analysis of the causes of low performance and the need for innovative strategies to improve results.

Table 1. Table of averages in the subject of mathematics.

| School year | course | Average over 10 points | No. of students |
|-------------|------------|------------------------|-----------------|
| 2021 - 2022 | PRMERO BGU | 7.38 | 265 |
| 2022 - 2023 | SECOND BGU | 7.52 | 240 |
| 2023 - 2024 | THIRD BGU | 9.08 | 177 |

Figure 2. Word cloud of the interview transcription document.



Impact on Learning and Motivation. The preliminary results obtained in the implementation of the project were highly

encouraging, evidencing significant improvements both in academic performance and in students' perception and motivation towards mathematics. These improvements were observed through an exhaustive analysis of quantitative and qualitative data collected during the different phases of the project, highlighting the effectiveness of the pedagogical strategies adopted.

Impact on Mathematical Logical Reasoning Skills. Post-tests designed to assess student progress revealed an average improvement of 30% in mathematical logical reasoning skills compared to the initial assessments. This progress was reflected in an increased ability to solve complex problems and apply mathematical concepts in practical contexts. Specifically, students showed stronger mastery in key areas such as:

- **Mathematical functions:** Understanding and handling of linear and quadratic functions, including the ability to represent them graphically and analyze their properties.
- **Analytical geometry:** Visualization and resolution of problems related to points, straight lines and geometric figures in the Cartesian plane.
- **Problem solving:** Development of systematic strategies to address mathematical challenges in a logical and structured manner.
- The use of GeoGebra played a key role in this progress, as it allowed students to interact dynamically with abstract concepts. The tool facilitated graphical visualization and direct experimentation, transforming learning from a passive to an active, exploratory process. For example, students were able to manipulate parameters in real time to observe how they affected the properties of functions, strengthening their conceptual understanding and their ability to make connections between algebraic and graphical representations.
- **Increase in Motivation and Attitude towards Mathematics.** At the qualitative level, students reported a positive change in their attitude towards mathematics, going from considering it a difficult and unmotivating subject to perceiving it as an interesting and accessible area. This change was captured through interviews and surveys, where students highlighted the following aspects:
- **Interactivity and dynamism in learning:** The incorporation of technological tools and active methodologies transformed

traditional classes into more attractive and participatory experiences.

- **Collaborative work:** Group activities fostered an environment of mutual support, where students learned from their peers and felt more confident in facing complex problems.
- **Practical relevance:** Activities linked to real-world situations helped students understand the usefulness of mathematics in their daily lives and future careers. In addition, many students noted that cooperative learning allowed them to improve their social and communication skills, developing greater confidence in themselves and their ability to solve problems. This inclusive approach also benefited students who had traditionally shown difficulties, who felt more integrated and supported in their learning process.
- **Teachers' Perception and Evaluation:** Teachers involved in the project expressed satisfaction with the tools and methodologies implemented, recognizing their positive impact on student learning and motivation. During the interviews, several teachers mentioned that GeoGebra not only facilitated the teaching of complex concepts, but also allowed them to innovate in their pedagogical practices. For example:
Facilitation of explanations: The ability to generate dynamic graphical representations improved the clarity of classroom explanations. **Adaptation to different learning styles:** The tool made it possible to cater to both visual learners and those who learn better through direct practice. However, some teachers noted the need for more extensive training to maximize the potential of educational technologies. While the initial workshops provided a solid foundation, teachers indicated that ongoing training would be key to address specific challenges and explore advanced functionalities of the tools.

The findings so far show that the project has not only improved students' academic performance, but also transformed their perception and attitude towards mathematics. This impact, supported by the integration of technologies such as GeoGebra and collaborative approaches, highlights the potential of innovative pedagogical strategies to overcome traditional barriers in teaching. Preliminary results also underscore the importance of ongoing teacher training to ensure the long-term sustainability and effectiveness of these initiatives. Overall, the project represents a

significant step towards creating more inclusive, dynamic and student-centered educational environments.

Discussion

By integrating the triangulation of sources, three axes related to institutional perception were identified: trends, tensions and absences, which synthesize the analysis of the training policy and allow conclusions and discussions on the categories set out in the questions related to conditions, institutional actions, foundations, key concepts and actors.

This project showed that pedagogical innovation can transform mathematics teaching, increasing both academic performance and student motivation. The combination of digital technologies and active methodologies, such as cooperative learning, represents an effective way to address current educational challenges.

To ensure the success and sustainability of these initiatives, it is recommended:

Extend the scope of the project to other educational levels and school contexts.

Establish continuous training programs for teachers, focused on the use of technologies and active methodologies.

Conduct long-term research to evaluate the impact of these strategies on students' academic performance and overall development.

The transformation of mathematics education is not only possible, but also necessary to prepare future generations for the challenges of the 21st century.

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